

Toward Bound-State Approach to Strangeness in Holographic QCD

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Abstract

An approach to realize a hyperon as a bound-state of a two-flavor baryon and a kaon is considered in the context of the Sakai-Sugimoto model of holographic QCD, which approach has been known in the Skyrme model as the bound-state approach to strangeness. As a simple case of study, pseudo-scalar kaon is considered as fluctuation around a baryon. In this case, strongly-bound hyperon-states are absent, different from the case of the Skyrme model. Observed is a weak bound-state which would correspond to $\Lambda(1405)$.

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1 Introduction

Chiral symmetry plays an important role in low energy of QCD, where the lightness of pions is because they could be the Nambu-Goldstone bosons of the spontaneous breaking of the symmetry. However, it is explicitly broken in *real* QCD by non-zero quark masses. In particular, the mass of the strange quark should be treated as large in contrast to those of the up and the down quarks. The original proposal of the Sakai-Sugimoto model [1, 2], which respects the chiral symmetry, is a holographic dual of massless QCD. Its quark-mass deformations have also been considered: A way is to use worldsheet instantons [3, 4] (see also [5, 6, 7])¹. In this paper, we focus on baryons with strangeness as a massive flavor in the Sakai-Sugimoto model.

Let us recall that, in the context of the Skyrme model, there is an approach known as the bound-state approach to strangeness [13, 14]. It is useful in the case that strangeness is dealt with as a massive flavor. The idea is to consider a kaon around a two-flavor Skyrmion, and then their bound-state is identified as a hyperon. There is a problem that they are somewhat strongly bound such that hyperon masses are lower than those obtained in experiments, but the approach seems to work well.

We would like to consider this idea in the Sakai-Sugimoto model, where baryons are identified as instanton-like solitons [1]. The soliton solution of the model has been studied in detail in [15] (see also [16, 17, 18, 19, 20, 21, 22, 23, 24, 25]). We focus on kaon fluctuation here as a simple attempt to the bound-state approach. This attempt should be different from the collective coordinate quantization of three flavor baryons [26], in which case mass shifts of baryon spectra in the presence of quark masses have been calculated in [27]². Our study may rather be related to meson-baryon scatterings such as \bar{K} - N scatterings.

This paper is organized as follows. In Section 2, we consider kaon fluctuation around the baryon as the background of the model. Here for simplicity we try to use a canonical kaon mass term. In Section 3, we consider the quark-mass term introduced by [3, 4]. This mass term should be a proper one to be used instead of the one used in Section 2, while we will see that its contribution is minor here. In Section 4, we summarize, and briefly discuss a possibility to take vector mesons into account.

¹ Another way is a tachyon condensation in the D8/ $\overline{\text{D8}}$ -branes [8, 9, 10, 11, 12].

² Baryon mass shifts in the case of two flavors have been studied in [28].

2 Kaon fluctuation around a baryon

Summary of the Sakai-Sugimoto model

The Sakai-Sugimoto model [1, 2] is a five-dimensional gauge theory, in which Kaluza-Klein(KK)-decomposed gauge fields are mesons and baryons are provided by its solitons.

The action is given by the following five-dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background:

$$S_{\text{SS}} = -\kappa \int d^4x dz \text{Tr} \left[\frac{1}{2} h(z) F_{\mu\nu}^2 + M_{\text{KK}}^2 k(z) F_{\mu z}^2 \right] + \frac{N_c}{24\pi^2} \int_{M_5} \omega_5. \quad (2.1)$$

The functions $h(z) = (1 + z^2)^{-1/3}$ and $k(z) = 1 + z^2$ inherit the metric of the curved background. The field strength is given by $F = dA + iA \wedge A$ with the $U(N_f)$ flavor gauge fields A_μ and A_z , and $\omega_5 = \text{Tr} \left(A F^2 - \frac{i}{2} A^3 F - \frac{1}{10} A^5 \right)$ is the Chern-Simons five-form. This model describes a holographic dual of a large N_c *massless* QCD.

There are two parameters in (2.1): M_{KK} and κ . For the mass scale M_{KK} , we chose a unit in which $M_{\text{KK}} = 1$. The dimensionful parameter can be easily recovered from dimensional analysis. The other is given by $\kappa = \lambda N_c / (216\pi^3)$ with the 't Hooft coupling constant λ . For later convenience we define $a \equiv 1/(216\pi^3)$ such that $\kappa = a\lambda N_c$. These are chosen as

$$M_{\text{KK}} = 949 [\text{MeV}], \quad \kappa = 0.00745, \quad (2.2)$$

to fit the experimental values³ of the ρ meson mass $m_\rho \simeq 776$ MeV and the pion decay constant $f_\pi \simeq 92.4$ MeV. We use (2.2) in this paper⁴.

A baryon is identified as an instanton-like soliton localized in the four-dimensional space x^α ($\alpha = 1, 2, 3, z$) [1]. The instanton number corresponds to the baryon number. Let us see the case that $N_f = 2$. A solution of the equations of motion of (2.1) has been obtained in [15] in the leading order in $1/\lambda$. The spatial component A_α^{inst} is precisely the $SU(2)$ BPST instanton, and also there is a nonzero $U(1)$ part A_0^{inst} ; in singular gauge, they are given by

$$\begin{aligned} A_i^{\text{inst}} &= -\frac{\rho^2}{\xi^2(\xi^2 + \rho^2)}((z - Z)\tau_i + \epsilon_{ijk}(x_j - X_j)\tau_k), \quad A_z^{\text{inst}} = \frac{\rho^2}{\xi^2(\xi^2 + \rho^2)}(x_i - X_i)\tau_i, \\ A_0^{\text{inst}} &= \frac{1}{16\pi^2 a \lambda} \frac{1}{\xi^2} \left[1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right] \mathbf{1}_2, \end{aligned} \quad (2.3)$$

³ If one rigorously treats the masslessness, numerical values in the chiral limit should be used for f_π and m_ρ .

⁴ One may fit the parameters using baryon data instead of using m_ρ . If the mass difference of Δ and N is used, $M_{\text{KK}} \sim 500$ MeV [15], and then κ can be calculated accordingly from f_π and M_{KK} .

where $\xi \equiv ((z - Z)^2 + |\vec{x} - \vec{X}|^2)^{1/2}$ is the distance in the spatial four dimensions, and τ_i ($i = 1, 2, 3$) are the Pauli matrices. The location of the soliton is $X^\alpha = (X^1, X^2, X^3, Z) = (\vec{X}, Z)$, and ρ is the instanton size. These correspond to five of the eight moduli of a $SU(2)$ instanton. Classically, the baryon is at the bottom of the curved space and has a finite size,

$$Z_{\text{cl}} = 0, \quad \rho_{\text{cl}}^2 = \frac{1}{8\pi^2 a \lambda} \sqrt{\frac{6}{5}}. \quad (2.4)$$

The non-zero value of ρ_{cl} is thanks to A_0^{inst} , which prevents the instanton shrinks to zero-size in the curved space. Thus A_0^{inst} is qualitatively important. In the following the baryon is placed at $\vec{X} = 0$.

Kaon fluctuation in the baryon background

Let us consider kaon fluctuation around the two-flavor baryon. Our strategy is such that we first reduce the five-dimensional action (2.1) to four-dimensional one which representing a kaon in a potential, and then consider bound-state problem following [13, 14]. In the following, we set $N_f = 3$.

The kaon can be turned on as a fluctuation of gauge fields around the two-flavor baryon as a instanton-like background gauge field. The two-flavor configuration (2.3) is embedded in 3×3 matrices, and fluctuation a_z , which is a two-component vector, is turned on at off-diagonal components,

$$A_\mu = \begin{pmatrix} A_\mu^{\text{inst}} & 0 \\ 0 & 0 \end{pmatrix}, \quad A_z = \begin{pmatrix} A_z^{\text{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}. \quad (2.5)$$

We assume that, even in the presence of the baryon as the background, a_z is decomposed into the four-dimensional kaon field as

$$a_z = -\frac{1}{\sqrt{2}} K(x^\mu) \phi_0(z), \quad K(x^\mu) = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad (2.6)$$

where we are using the gauge in which the KK-modes with $n \geq 1$ are gauged away. The factor in front of $K(x^\mu)$ corresponds to the canonical normalization of $K(x^\mu)$. The zero-mode ϕ_0 is chosen such that

$$\phi_0(z) = \frac{1}{\sqrt{\kappa\pi}} \frac{1}{k(z)}, \quad \kappa \int_{-\infty}^{\infty} dz k(z) \phi_0(z)^2 = 1. \quad (2.7)$$

To be precise, (2.6) and (2.7) were obtained when quarks are massless and baryons are absent [1]. We believe that the reduction to four dimensions by using these would be a convenient assumption and approximation.

A four-dimensional action of $K(x^\mu)$ gives a starting point of our bound-state approach. Putting (2.5) into (2.1), and keeping the terms quadratic in a_z , we obtain

$$S_{\text{fluc}} = -2\kappa \int d^4x dz k(z) \bar{D}_\mu^{(\text{inst})} a_z^\dagger D^{(\text{inst})\mu} a_z, \quad (2.8)$$

where the covariant derivatives are given by

$$D_\mu^{(\text{inst})} a_z = \partial_\mu a_z + i A_\mu^{\text{inst}} a_z, \quad \bar{D}_\mu^{(\text{inst})} a_z^\dagger = \partial_\mu a_z^\dagger - i a_z^\dagger A_\mu^{\text{inst}}. \quad (2.9)$$

To do the integral over z , we use (2.7), and then obtain

$$S_{\text{kaon}} = - \int d^4x \left[\partial_\mu K^\dagger \partial^\mu K + i \left(\partial^\mu K^\dagger \Psi_\mu K - K^\dagger \Psi_\mu \partial^\mu K \right) + K^\dagger \Upsilon K \right], \quad (2.10)$$

where

$$\Psi_\mu \equiv \frac{1}{\pi} \int dz \frac{1}{k(z)} A_\mu^{\text{inst}}, \quad \Upsilon \equiv \frac{1}{\pi} \int dz \frac{1}{k(z)} (A_\mu^{\text{inst}})^2. \quad (2.11)$$

Note that Ψ_i is written in Pauli matrices, while Ψ_0 and Υ are proportional to unit matrices,

$$\begin{aligned} \Psi_i &= -\Theta^{(1)} \epsilon_{ijk} x_j \tau_k, & \Psi_0 &= \frac{1}{16\pi^2 a \lambda} \Theta^{(2)} \mathbf{1}_2, \\ \Upsilon &= \left[3\Theta^{(3)} + 2r^2 \Theta^{(4)} - \left(\frac{1}{16\pi^2 a \lambda} \right)^2 \Theta^{(5)} \right] \mathbf{1}_2, \end{aligned} \quad (2.12)$$

where $\Theta^{(n)}$ are given in Appendix. It is convenient to define $\Psi_0 \equiv \hat{\Psi}_0 \mathbf{1}_2$ and $\Upsilon \equiv \hat{\Upsilon} \mathbf{1}_2$.

Note that (2.3) is a flat-space instanton. This is obtained in $z \ll 1$ region, where the space is almost flat $k(z), h(z) \sim 1$. Effects of the curved space come in at large z region. However, in the present case we are interested in the overlap of ϕ_0 and the baryon, both of which are localized at $z = 0$. To use (2.3) would be a meaningful approximation.

Equation of motion

Recall that (2.10) corresponds to massless kaon. We therefore need a mass term. In this section we try to use the canonical kaon mass term for simplicity,

$$S_{\text{kaon}} - \int d^4x m_K^2 K^\dagger K, \quad (2.13)$$

where $m_K \simeq 495$ MeV. We will consider a quark mass term which is precisely introduced to the Sakai-Sugimoto model separately in the next section.

Since we have the four-dimensional action (2.10), we may simply trace the analysis of [13, 14]. We decompose the kaon field by creation and annihilation operators as⁵

$$K(x^\mu) = \sum_{n>0} \left(k_n(r) Y_{TT_z L}(\Omega_2) e^{i\omega_n t} a_n + \tilde{k}_n(r) Y_{TT_z L}(\Omega_2) e^{-i\tilde{\omega}_n t} b_n^\dagger \right), \quad (2.14)$$

where $Y_{TT_z L}$ is spherical harmonics on S^2 . The basis of the harmonics is labeled by angular momentum \mathbf{L} and total spin $\mathbf{T} = \mathbf{I} + \mathbf{L}$, where $\mathbf{I} = \boldsymbol{\tau}/2$.

After a short calculation, we obtain the following eigenvalue equations,

$$\left[\frac{1}{r^2} \partial_r (r^2 \partial_r) + \omega_n^2 + 2\hat{\Psi}_0 \omega_n - V(r) \right] k_n(r) = 0, \quad (2.15)$$

$$\left[\frac{1}{r^2} \partial_r (r^2 \partial_r) + \tilde{\omega}_n^2 - 2\hat{\Psi}_0 \tilde{\omega}_n - V(r) \right] \tilde{k}_n(r) = 0, \quad (2.16)$$

where

$$V(r) = \frac{L(L+1)}{r^2} + 4\Lambda_{\mathbf{I} \cdot \mathbf{L}} \Theta^{(1)} + \hat{\Upsilon} + m_K^2, \quad (2.17)$$

with the eigenvalue of the spin-isospin interaction given by

$$\mathbf{I} \cdot \mathbf{L} Y_{TT_z L}(\Omega_2) = \frac{T(T+1) - L(L+1) - 3/4}{2} Y_{TT_z L}(\Omega_2) \equiv \Lambda_{\mathbf{I} \cdot \mathbf{L}} Y_{TT_z L}(\Omega_2). \quad (2.18)$$

As seen from the signs in front of $\hat{\Psi}_0$, (2.15) and (2.16) correspond to $S = -1$ (normal) and $S = +1$ (exotic) channels, respectively. Below we focus on (2.15). In the Skyrme model, the exotic channel was studied in the context of penta-quarks [29]. Note that, these equations are somehow different from those obtained in [13, 14]. Firstly, since the kinetic term of (2.10) is canonical, no additional functions of r appear in ∂_r and ω terms. Secondly, we will see that the behavior of $V(r)$ is repulsive to dislike bound-states.

We shall consider two cases of (T, L) : $T = \frac{1}{2}, L = 1$ and $T = \frac{1}{2}, L = 0$. In [13, 14], the former case corresponds to Λ , Σ and Σ^* , while the latter case did $\Lambda(1405)$; for details, see the papers. Such identification was because the bound-states could be found, and the spectra were nicely interpreted as those baryons. Relying on the observations, we now search whether there are bound-states in such two cases.

Solving the eigenvalue problems, we find no bound-state in the $T = \frac{1}{2}, L = 1$ case, while a weak bound-state can be found in the $T = \frac{1}{2}, L = 0$ case with $\omega_1 \sim 490$ MeV. In the former case, $V(r)$ is strongly repulsive opposite to the Skyrme model case, where the potential are strongly attractive and favor a bound-state. In the latter case, the potential

⁵ The signs of the exponents are different from [13, 14] due to our convention.

is still repulsive (see Figure 2). However, in practice Ψ_0 in (2.15) gives seemingly large contribution which forces kaon wavefunction localize at the slight minimum of $V(r)$ in this case. We concern that this may be a numerical artifact of the baryon solution of the Sakai-Sugimoto model because A_0^{inst} , which should have been of order $\frac{1}{\lambda}$ and small, is in fact large due to the numerical factor appearing in it.

It seems that our bound-state program tries to bind a kaon and a baryon, both of which have concrete identity after that they are made from a quark-antiquark pair or N_c quarks. This point may be related to a prescription where $\Lambda(1405)$ can be a bound-state of \bar{K} and N .

We do not step in the $SU(2)$ corrective coordinate quantization of the baryon [15] although the calculation is straightforward. It will produce the mass splittings of Λ , Σ and Σ^* baryons, for example. Since we could not obtain any bound-state corresponding in $T = \frac{1}{2}, L = 1$ case, such mass splittings cannot be discussed.

3 Kaon mass term revisited

In the previous section, the canonical kaon mass term $m_K^2 K^\dagger K$ was employed. However, there are precise ways to introduce quark masses by using ingredients of string theory. In this section, we consider the quark mass term introduced in [3]. We will see that its contribution is minor in the present case of the bound-state problem. For this reason we used $m_K^2 K^\dagger K$ phenomenologically in the previous section.

The quark-mass term introduced in [3] is

$$\delta S = c \int d^4x \text{PTr} \left[M \left(\exp \left[-i \int_{-z_m}^{z_m} A_z dz \right] - \mathbf{1}_{N_f} \right) \right] + \text{c.c.} \quad (3.1)$$

This term can be induced from worldsheet instanton amplitude in the case that D6-branes are put into the D-brane configuration of the Sakai-Sugimoto model. Here c is a constant which relates quark masses to meson masses. We leave it undetermined, but we will soon rewrite the mass term in terms of the meson masses. Because (3.1) is introduced as perturbation from the chiral limit, z_m must be close enough to $z = \infty$, and hence we may approximate the integral region as

$$\delta S = c \int d^4x \text{PTr} \left[M \left(\exp \left[-i \int_{-\infty}^{\infty} A_z dz \right] + \exp \left[i \int_{-\infty}^{\infty} A_z dz \right] - 2 \mathbf{1}_{N_f} \right) \right]. \quad (3.2)$$

For simplicity, we set $m_u = m_d = m_0$ such that $M = \text{diag}(m_0, m_0, m_s)$. Below we derive a kaon mass term in the baryon background.

The path-ordering with respect to the background A_z^{inst} is abelian. This is because A_z^{inst} is proportional only to a matrix $\vec{x} \cdot \vec{\tau}$ [30]. Thus the path-ordering reduces to an ordinary integral,

$$\text{P exp} \left[-i \int_a^b dz \begin{pmatrix} A_z^{\text{inst}} & 0 \\ 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} \exp \left[-i \int_a^b A_z^{\text{inst}} dz \right] & 0 \\ 0 & 1 \end{pmatrix}. \quad (3.3)$$

For convenience, we use a notation

$$\int_a^b dz A_z^{\text{inst}} = I_{(a,b)} \hat{x}^i \tau_i, \quad (3.4)$$

where $\hat{x}_i = x_i/r$ are unit vectors ($r = |\vec{x} - \vec{X}|$), and

$$I_{(a,b)} = \left[\tan^{-1} \left(\frac{b}{r} \right) - \frac{r}{\sqrt{r^2 + \rho^2}} \tan^{-1} \left(\frac{b}{\sqrt{r^2 + \rho^2}} \right) \right] - (b \rightarrow a). \quad (3.5)$$

However, the path-ordering is no longer abelian if a_z is turned on. Putting (2.5) in (3.2) and keeping terms of order $a_z^\dagger a_z$, we obtain

$$\begin{aligned} S_{\text{mass}} = & - \int d^4x \left[f_\pi^2 \left(m_K^2 - \frac{m_\pi^2}{2} \right) \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{z_2} dz_1 \cos(I_{(z_1, z_2)}) a_z^\dagger(z_1) a_z(z_2) \right. \\ & \left. + \frac{f_\pi^2 m_\pi^2}{2} \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{z_1} dz_2 \cos(I_{(-\infty, \infty)} - I_{(z_2, z_1)}) a_z^\dagger(z_2) a_z(z_1) \right], \end{aligned} \quad (3.6)$$

where the constant c and the quark masses are rewritten in terms of the meson masses [27],

$$m_\pi^2 = \frac{4c}{f_\pi^2} m_0, \quad m_K^2 = \frac{2c}{f_\pi^2} (m_0 + m_s). \quad (3.7)$$

The path-ordering is almost abelian where two fluctuations a_z and a_z^\dagger are inserted. The double-integrals in (3.6) are with respect to the positions of them.

We can obtain an effective kaon mass term after reducing (3.6) to four dimensions. We have

$$S_{\text{mass}} = - \int d^4x M_{\text{eff}}^2(r) K^\dagger(x^\mu) K(x^\mu), \quad (3.8)$$

where the effective kaon mass is a function of r ,

$$\begin{aligned} M_{\text{eff}}^2(r) = & \frac{2}{\pi^2} \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{z_2} dz_1 \frac{1}{k(z_1)k(z_2)} \\ & \times \left[\left(m_K^2 - \frac{m_\pi^2}{2} \right) \cos(I_{(z_1, z_2)}) + \frac{m_\pi^2}{2} \cos(I_{(-\infty, \infty)} - I_{(z_1, z_2)}) \right]. \end{aligned} \quad (3.9)$$

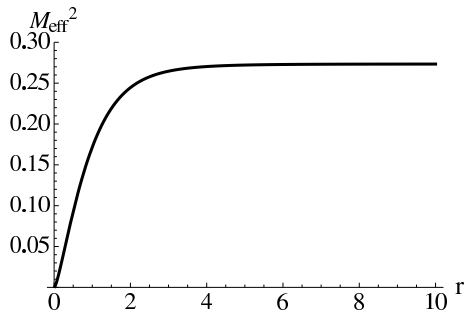


Figure 1: Behavior of $M_{\text{eff}}^2(r)$. $M_{\text{eff}} = m_K$ at $r = \infty$ while $M_{\text{eff}} = 0$ at $r = 0$.

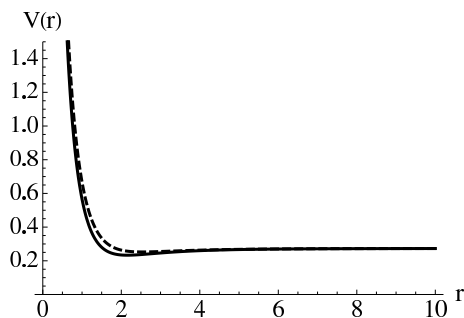


Figure 2: $V(r)$ (dashed), and its counterpart if M_{eff} is used instead of m_K (solid).

The behavior of M_{eff} is plotted in Figure 1. Here we set $m_0 = 0$ for simplicity. M_{eff} goes to m_K away from the baryon, while it vanishes sufficiently at the center of the baryon. Note that the baryon is now given by a finite-size soliton.

In the case that (3.9) is used instead of m_K^2 , the potential (2.17) is slightly modified as shown in Figure 2. The effect is minor since the region where M_{eff}^2 drops is where $V(r)$ grows up⁶. The bound-state energy in the $T = \frac{1}{2}, L = 0$ case is not largely modified: It decrease only a few MeV. There is still no bound-state in the $T = \frac{1}{2}, L = 1$ case.

4 Summary and discussion

Toward the bound-state approach to strangeness in the context of the Sakai-Sugimoto model of holographic QCD, we considered, as a simple case of study, fluctuation which corresponds to kaon around the instanton-like baryon. We then used a four-dimensional action reduced from five-dimensional one. Suggested by analyses in the Skyrme model [13, 14], we focused on two cases which characterized the kaon wavefunction: the $T =$

⁶ If some contribution, as will be discussed in section 4, drastically changes the repulsive behavior of the potential, then the effect of the reduce of M_{eff}^2 at small r will be significant.

$\frac{1}{2}$, $L = 1$ case and the $T = \frac{1}{2}$, $L = 0$ case. In both cases, the potential which the kaon feels were repulsive. There was no bound-state in the former case, while a weak bound-state was seen in the latter case although this might be a numerical artifact of the baryon solution of the Sakai-Sugimoto model. This would correspond to a $\Lambda(1405)$ described by a meson-baryon bound-state.

The disparity of ours from the case of the Skyrme model is in a sense apparent. In the latter case, the kaon fluctuation is smoothly connected to the $SU(3)$ collective coordinate quantization of the Skyrmion in the chiral limit. However, as long as focusing on kaon fluctuation as in (2.5), the collective coordinate quantization of the three-flavor baryon [26] should not be realized because all gauge field fluctuations need to be included so as to generate the instanton zero-mode.

If we want to find connections to collective coordinate quantization description, it may be necessary to include also vector-mesons. In fact, a work of bound-state approach in hidden local symmetry framework [31], which work contained a vector-meson counterpart of kaon, could give a result similar to the case of the Skyrme model. In this line, there is another work which considered vector-mesons of pions but that of the kaon is absent [32]. The latter may be similar to our attempt.

However, dealing with vector-mesons in the case here will be far more difficult for following reasons. First, quark masses should be included in a way that mass spectra of vector-mesons can be calculated in the presence of them. Secondly, effective vector meson masses should be calculated in baryon background as in Section 3. To obtain an effective low-energy description which will be in the same universality class of the bound-state approach in the Skyrme model, all the vector mesons (of kaons) should be integrated out at some energy scale. It is a question whether this story really works.

There is a gauge so-called “ $A_z = 0$ gauge” in the Sakai-Sugimoto model. In this gauge, the Skyrme model action can be obtained from the Sakai-Sugimoto model; pion field appears from non-normalizable mode at the boundary of the curved space. But baryons are located at the bottom of it. Hence, to study meson-baryon systems we should know the overlap of these two. Using this gauge may be in direct connection to the bound-state approach in the Skyrme model, but whether this is the case or not remains to be seen.

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A Appendix

The following symbols are used for convenience:

$$\Theta^{(1)} \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{1}{1+z^2} \frac{\rho^2}{\xi^2(\xi^2 + \rho^2)}, \quad (\text{A.1})$$

$$\Theta^{(2)} \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{1}{1+z^2} \frac{1}{\xi^2} \left(1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right), \quad (\text{A.2})$$

$$\Theta^{(3)} \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{1}{1+z^2} (z - Z)^2 \left(\frac{\rho^2}{\xi^2(\xi^2 + \rho^2)} \right)^2, \quad (\text{A.3})$$

$$\Theta^{(4)} \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{1}{1+z^2} \left(\frac{\rho^2}{\xi^2(\xi^2 + \rho^2)} \right)^2, \quad (\text{A.4})$$

$$\Theta^{(5)} \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} dz \frac{1}{1+z^2} \left[\frac{1}{\xi^2} \left(1 - \frac{\rho^4}{(\rho^2 + \xi^2)^2} \right) \right]^2. \quad (\text{A.5})$$

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